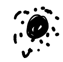
Multi-particle Continued



← C.O.M

The motion quantities $\boxed{\vec{L}, \vec{H}, E}$

Linear momentum

$$\vec{L} = \sum m_i \vec{v}_i \underset{\text{simplification}}{=} \frac{d}{dt}(m_{tot} \vec{r}_G)$$

Any system $\underline{\underline{\text{is}}}$ a particle

$$\vec{F}^{Tot} = m\vec{a}_G$$

Angular Momentum $\vec{H}$

-always referenced to a point in space

$$\vec{H}_{/c} = \sum \vec{r}_{i/c'} \times m\vec{v}$$

c' is fixed in F and instantaneously coinciding with c

We need c' for derivative to make sense

$$\vec{M}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{a}_{i/F} = \dot{\vec{H}}_{/c}$$

Useful fact: $\sum \vec{M}_{/G} = \vec{H}_{/G}$

$$\vec{H}_{/G} = \sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}$$ → favorite in freshman physics

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$\vec{H}_{/c} = \frac{d}{dt}[\text{nothing}]$ → there is no parallel to linear momentum $= m\frac{d}{dt}r$

- falling cat

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$$\vec{H}_{/c} = \underbrace{\vec{r}_{G/c} \times m_{tot}\vec{v}_{G/c}}_{\vec{H}_{G/c}} + \underbrace{\sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}}_{\vec{H}_{/G}}$$

$$\vec{M}_{/c} = \dot{\vec{H}}_{G/c} + \dot{\vec{H}}_{/G}$$

$$\vec{r}_{G/c} \times \vec{F} + \vec{M}_{/G} = \dot{\vec{H}}_{G/c} + \dot{\vec{H}}_{/G}$$

$$\vec{r}_{G/c} \times \vec{F}^{tot} = \dot{\vec{H}}_G \qquad \vec{M}_{/G} = \dot{\vec{H}}_{/G}$$

Kinetic Energy

Konig's Theorem

$$E_K = \frac{1}{2} m_{tot} V_G^2 + \underbrace{\frac{1}{2} \sum m_i |(\vec{V}_i - \vec{V}_G)|^2}_{\frac{1}{2} \sum m_i \vec{V}_{i/G} \cdot \vec{V}_{i/G}}$$

can't break into two simple terms

$$\underbrace{\vec{F}^{Tot} \cdot \vec{V}_G}_{\text{power}} = \frac{d}{dt} E_{GK} = \frac{d}{dt} \frac{1}{2} m V_G^2$$

dimensions of power, but is not the real power of the forces

Matlab Examples:

(see 2030 course Matlab samples)